Detailed and Integrated Representation of Spatial Relations

WU Changbin, Chen Xia

College of Geographical science Nanjing Normal University, Nanjing 210023, China

Abstract. Spatial relation is one of the basic topics of geographic information systems (GISs), and it has been widely applied in data organization and spatial analysis. It seemed whether single topological, metric or directional relation can hardly represent complex spatial relations in some cases. Different topological relations may result in the same expression by some topological relation models, among which 9-Intersection Model (9IM) and its extended models (e.g. DE+9IM) are well known. However, we are used to describe spatial relation in several parts to make it more accurate, e.g., “Go north from here and along the lake about 200 meters, and then you will arrive.” In this paper, the detailed and integrated model of the topological, metric and directional relation is proposed, to describe the complex spatial relations between region and region, line and line, line and region more exactly. We use one dimensional matrix to represent topological and directional relations, and two dimensional matrixes to represent metric relation. It’s better to answer the question, e.g., “How the road crosses the park?” We also give an application finally, to calculate the actual area of the land use parcels, from which the area of related linear features is to be subtracted, according to their topological relations in details.

Key words: spatial relation, topological relation, 9-intersection model

1. Introduction

Spatial relation is one of the basic topics of geographic information systems (GISs), and it has been widely applied in data organization and spatial analysis. Spatial relation includes topological relation, metric relation and directional relation. Many scholars have focused on the progresses of spatial relation researches, among which the topological relations are studied mostly. Egenhofer *et al.* (1991, 1993) described 9-Intersection model (9IM), and other extended models by 9IM such as DE+9IM, CBM (Clementini 1993, 1995), V9IM (Chen *et al.* 2000) ， Three-Valued 9-Intersection Model (Kurata 2009a, 2010). Randell *et al.* (1992) proposed Region Connection Calculus (RCC), which uses connection *C(x, y)* to describe relations between objects. Chang *et al.* (1987) took 2D-String to detect topological relations between objects according to their projections on *X* and *Y* axis. About metric relations, Egenhofer *et al.* (1998), Shariff *et al.* (1998) used *splitting* to determine how a region’s interior, boundary, and exterior are divided by a line’s interior, boundary, and vice versa; *closeness* measures to describe how far apart disjoint parts

are. About directional relation, the main methods as follows: Minimum Boundary Rectangle (MBR) (Papadias *et al.* 1996), CONE (Haar 1976), 2D-String (Chang *et al.* 1987) and etc.

And in recent years, Duboisset *et al.* (2007)，Egenhofer(2009), Schneider *et al.* (2004,2006), Praing *et al.* (2008), Mckenney *et al.* (2006,2007),Kurata(2008),Zhilin *et al.* (2006), Kimfung *et al.* (2007) discussed the complex topological relations between composite features.

There also are some scholars who integrated topological, metric and directional relation (Sharma 1996, Hong 1996, GODOY *et al.* 2004, Cicerone *et al.* 2003, Schwering 2007 ,Dube *et al.* 2009, Kurata 2009b), but they mostly paid attention to interactive reasoning of these three relations instead of their integrated representation. Actually, in practice, we often describe these three relations together, e.g., “How long and where does this tunnel cross the hill? How long does the road pass along the lake?” We are used to describe spatial relations in several parts to make it more accurate, but not simply or equivocally. We may say, e.g., “Go north from here and along the lake about 200 meters, then you will arrive.” Therefore, it tends to be more in accordance with our lives when the topological relation, metric relation and directional relation are integrated together.

Spatial relations between point and point, point and line, point and region are seemed to be simple, but those between region and region, line and line, line and region are more complex. So we here only aim at these complex relations, to make out their detailed and integrated representation, suppose our objects are polylines without arcs, but regions can be convex or concave.

The remainder of this paper is structured as follows: *Section 2* proposes one general integrated method to describe topological, metric and directional relation. *Section 3* takes some examples of spatial relations between region/region, line/line, and line/region to prove the method above. *Section 4* shows the applications of our research, for an example that how to calculate the actual area of land use parcels according to the detailed topological relations. *Section 5* draws some conclusions and discusses future works.

2. Integrated representation of spatial relations

If *R (A, B)* represents the composite spatial relation between *A* and *B*, it can be split into *n* parts of single spatial relations, which are $R\ (A_1, B_1)$, $R\ (A_2, B_2)$… $R\ (A_n, B_n)$:

$$R\ (A, B) = \begin{bmatrix} \mathrm{R}(\mathrm{A}_1, \mathrm{B}_1) \\ \mathrm{R}(\mathrm{A}_2, \mathrm{B}_2) \\ \cdots \\ \mathrm{R}(\mathrm{A}_n, \mathrm{B}_n) \end{bmatrix} \qquad (1)$$

If we use *T*, *M*, *D* to describe respectively the topological relation, metric relation, and the directional relation between *A* and *B*, then *R (A, B)* can be represented as follows:

$$R(A,B)=T(A,B)\vee M(A,B)\vee D(A,B) \qquad (2)$$

Whereas $T(A_1, B_1)$, $T(A_2, B_2)$,…,$T(A_n, B_n)$ are serial topological relations between (A_1,B_1), (A_2,B_2),…, (A_n, B_n), and $D(A_1,B_1)$, $D(A_2,B_2)$,… ,$D(A_n, B_n)$ are serial directional relations; suppose there are k kinds of metric relations(e.g., length, area) between A and B, recorded by $M1, M2,\ldots,Mk$, then we get $M1_1, M1_2,\ldots, M1_n$ to describe the first metric relations between $(A_1, B_1),(A_2, B_2),\ldots,(A_n, B_n)$, and $Mk_1, Mk_2,\ldots, Mk_n$ to describe the k kind metric relations, see *Figure 1*.

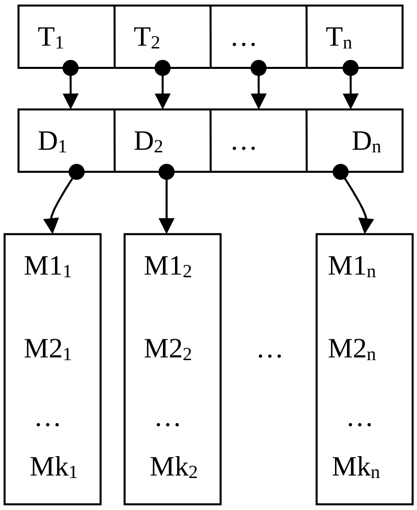

Figure 1. Topological, metric, directional relations integrated model

We use one dimensional matrix to describe topological and directional relations, and two dimensional matrixes to describe metric relation, as Eq. 3:

$$R\ (A, B) = \begin{bmatrix} T(A_1,B_1) \\ T(A_2,B_2) \\ \cdots \\ T(A_n,B_n) \end{bmatrix} \vee \begin{bmatrix} M_1(A_1,B_1) & M_2(A_1,B_1) & \cdots & Mk(A_1,B_1) \\ M_1(A_2,B_2) & M_2(A_2,B_2) & \cdots & Mk(A_2,B_2) \\ \cdots & \cdots & \cdots & \cdots \\ M_1(A_n,B_n) & M_2(A_n,B_n) & \cdots & Mk(A_n,B_n) \end{bmatrix} \vee \begin{bmatrix} D(A_1,B_1) \\ D(A_2,B_2) \\ \cdots \\ D(A_n,B_n) \end{bmatrix} \qquad (3)$$

3. Examples

3.1. Relations between region and region

According to studies of Egenhofer, there are 8 basic topological relations between region and region, i.e., *disjoint, meet, equal, inside, contains, covers, covered by, overlap*. But in *Figure 2*, the topological relation between A and B can hardly be described by "*overlap*". However, it can be split into two parts, to get $R\ (A, B) = R\ (A_1, B_1) + R\ (A_2, B_2)$. If only the intersection area is considered as their metric measure, then we get:

$$R\ (A, B) = \begin{bmatrix} T(A_1,B_1) \\ T(A_2,B_2) \end{bmatrix} \vee \begin{bmatrix} Area(A_1 \cap B_1) \\ Area(A_2 \cap B_2) \end{bmatrix} \vee \begin{bmatrix} D(A_1,B_1) \\ D(A_2,B_2) \end{bmatrix} \qquad (4)$$

In which: $T\ (A_1, B_1) = T\ (A_2,B_2)=\begin{bmatrix} \phi & \phi & \neg\phi \\ \phi & \neg\phi & \neg\phi \\ \neg\phi & \neg\phi & \neg\phi \end{bmatrix}$;

Suppose $Area\ (A_2 \cap B_2)$ =100m^2, $D\ (A_1, B_1) = D\ (A_2, B_2)$ =North.

The natural language of spatial relation between *A* and *B* as: *A* is in the north of *B*; *A* is partly overlapped and adjoined by *B*, the intersection area of which is 100m^2.

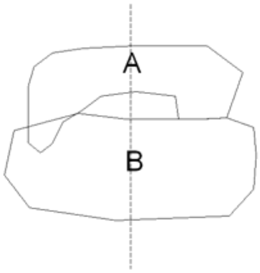

Figure 2. Spatial relationship between complex regions

3.2. Relations between line and line

As shown in *Figure 3*, it's really complex for the relation between *A* and *B*, because both intersection and overlay exist. Now this complex relation can be structured in three parts, i.e., $R\ (A, B) = R\ (A_1, B_1) + R\ (A_2, B_2) + R\ (A_3, B_3)$. Length is the metric measure to illuminate how long these two lines overlap. Directional relation is not considered here, since people seldom care for it.

$$R\ (A, B) = \begin{bmatrix} T(A_1, B_1) \\ T(A_2, B_2) \\ T(A_3, B_3) \end{bmatrix} \vee \begin{bmatrix} Len(A_1 \cap B_1) \\ Len(A_2 \cap B_2) \\ Len(A_3 \cap B_3) \end{bmatrix} \qquad (5)$$

In which: $T\ (A_1, B_1) = T\ (A_2,B_2)=\begin{bmatrix} \phi & \phi & \neg\phi \\ \phi & \phi & \neg\phi \\ \neg\phi & \neg\phi & \neg\phi \end{bmatrix}$ $T\ (A_2,B_2)=\begin{bmatrix} \neg\phi & \phi & \neg\phi \\ \phi & \phi & \neg\phi \\ \neg\phi & \neg\phi & \neg\phi \end{bmatrix}$

Suppose :$Len(A_1,B_1)$=100m, $Len(A_2,B_2) = Len(A_3,B_3)$=0

The natural language of spatial relation between *A* and *B* as: Line *A* is partly overlapped by *B* for 100m, and then disjointed from *B*, finally they intersect at one point.

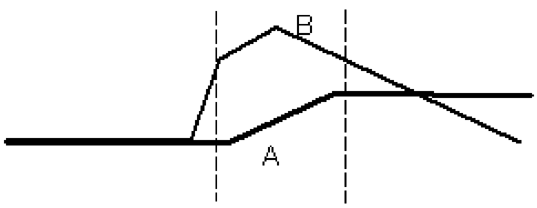

Figure 3. Spatial relationship between complex polylines

3.3. Relations between line and region

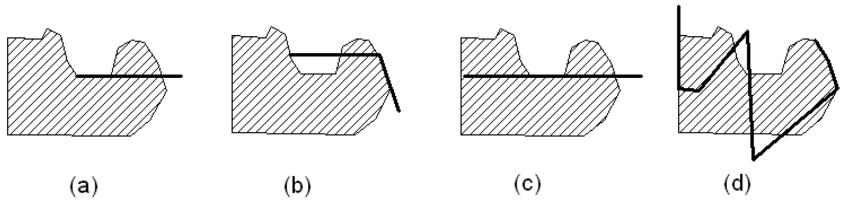

Figure 4. Four different topological relationships between line and region but similarly represented by 9IM and DE+9IM

As shown in *Figure 4*, there are four different relations between a line and a region. It seems to lines all pass through the outside, boundary, and inner of regions, i.e., lines all “*cross*” regions; even more, the expressions of 9IM and DE+9IM are the same (Eq.6, Eq.7).

$$T\ (A, L) = \begin{bmatrix} A^{\circ} \cap L^{\circ} & A^{\circ} \cap \partial L & A^{\circ} \cap L^{-} \\ \partial A \cap L^{\circ} & \partial A \cap \partial L & \partial A \cap L^{-} \\ A^{-} \cap L^{\circ} & A^{-} \cap \partial L & A^{-} \cap L^{-} \end{bmatrix} = \begin{bmatrix} \neg\phi & \phi & \neg\phi \\ \neg\phi & \neg\phi & \neg\phi \\ \neg\phi & \neg\phi & \neg\phi \end{bmatrix} \qquad (6)$$

$$Dim\ (A, L) = \begin{bmatrix} \dim(A^{\circ} \cap L^{\circ}) & \dim(A^{\circ} \cap \partial L) & \dim(A^{\circ} \cap L^{-}) \\ \dim(\partial A \cap L^{\circ}) & \dim(\partial A \cap \partial L) & \dim(\partial A \cap L^{-}) \\ \dim(A^{-} \cap L^{\circ}) & \dim(A^{-} \cap \partial L) & \dim(A^{-} \cap L^{-}) \end{bmatrix} = \begin{bmatrix} 1 & - & 2 \\ 1 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \qquad (7)$$

Indeed, there are obvious differences in the order, number, and length of the crossing sections.

For any polyline *A* and region *B*, when they intersect we get *n* nodes. So *L* may split into *n*-1 straight lines, recorded by $L_1, L_2, \ldots, L_{n-1}$.Thus, there are only three classes for the inner line L_i : a)$L_i^{\circ} \subset A^{-}$, (i.e., the line holy outside of the region); b) $L_i^{\circ} \subset \partial A$, (i.e., the line holy at the boundary of the region); c) $L_i^{\circ} \subset A^{\circ}$, (i.e., the line in the region). Now it would not happen that the line partly lay outside of the region and partly in the region. We can get 6 classes of topological relations, their semantics are: *inside, meet, disjoin, out touch, in touch, overlap*.

Taking *Figure 4* (a) as an example, if *L* is a road, *A* is a park, then we get three parts after intersecting, suppose their lengths are 200m, 300m, 100m. Thus the topological relation between the road and park can be expressed by: $R\ (A, L) = R\ (A, L_1) + R\ (A, L_2) + R\ (A, L_3)$.

$$R\ (L, A) = \begin{bmatrix} T(A, L_1) \\ T(A, L_2) \\ T(A, L_3) \end{bmatrix} \vee \begin{bmatrix} Len(L_1) \\ Len(L_2) \\ Len(L_3) \end{bmatrix} = \begin{bmatrix} meet \\ in \cdot touch \\ out \cdot touch \end{bmatrix} \vee \begin{bmatrix} 200 \\ 300 \\ 100 \end{bmatrix} \qquad (8)$$

The natural language of spatial relation between *A* and *B* as: This road firstly lies in the boundary of the park for 200m; then partly in the park for 300m; finally outside of the park for 100m. As a result, now we can know clearly “how does this road pass through the park (Mark and Egenhofer, 1994)?”

4. Applications

In Chinese land use database, a continual region which has the same land use class is mapped as a parcel, and a zonal land use class is surveyed by its sketch line and mapped as a linear feature, such as a small road or channel within the farm. Let *GS* be the parcel's geometrical area, and *LS* be the total area of the linear feature within the parcel , then the parcel's actual area (*AS*) should be calculated as follows: *AS*=*GS*-*LS*. As shown in *Figure 5 (a)*, a farm is passed by a long channel, which has just a short width (e.g., 2 meter). But on the small scale map (such as 1:10000) the channel is represented as a single line *l* which has no area of geometry (*Figure 5 (b)*). In the case, rather than the parcel's geometrical area, the actual area of parcel *p* should be reduced by the related area of channel within the farm, see the shadow part in *Figure 5 (a)*.

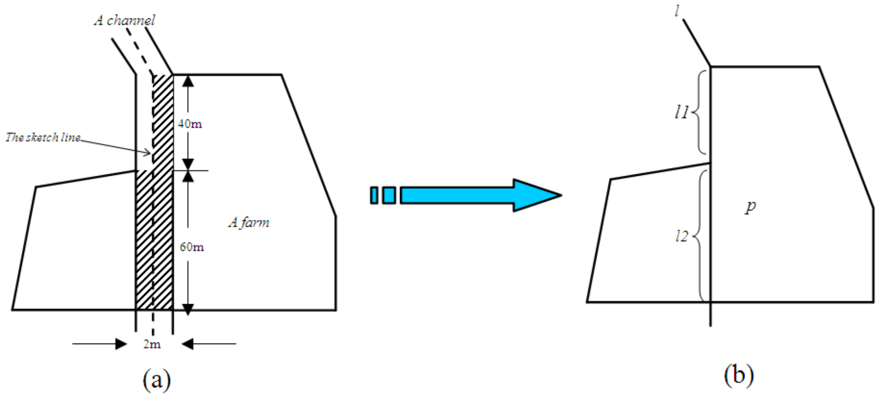

Figure 5. (a)A channel passed a farm in reality; (b) the channel drawn as a linear feature on the small scale map (such as 1:10000)

We distinguish the different topological and metric relations between the parcel and linear feature in details, so as to accurately calculate how much area of the linear feature within the parcel to be subtracted from (i.e. *LS*).

a) If the part of linear feature is outside of the parcel, its area need not to be subtracted;
b) If the part of linear feature is entirely within the parcel, its all area should be subtracted;
c) If the part of linear feature is along the boundary of the parcel, its area should be subtracted half.

As shown in *Figure* 5(b), the line *l* can be divided by some parts *l1*, *l2* and etc. *l2* is entirely within (but *in touch*) the parcel *p*, so all the area of *l2* (120m^2=60m×2m) has to be subtracted from *p*; *l1* passes along (*meet*) the boundary of *p*, so half of the area of *l1* (40m^2=40m×1m) to be subtracted; the other part of *l* is outside of *p*, so needn't to be subtracted.

$$R\ (p,l) = \begin{bmatrix} \mathrm{T}(p,l1) \\ \mathrm{T}(p,l2) \end{bmatrix} \vee \begin{bmatrix} Area(l1) \\ Area(l2) \end{bmatrix} = \begin{bmatrix} meet \\ in\ touch \end{bmatrix} \vee \begin{bmatrix} 40 \\ 120 \end{bmatrix}$$

We developed a program to calculate the parcels' actual area, which passed by some linear features, the attribute lists show the results of actual area, as shown in

Figure 6. *Figure 7* shows the records of metric relations, according the detailed topological relations between the parcels and linear features, whereas *DKH* means the parcel *ID*, *XZBH* means the linear *ID* related to the parcel, *XZCD* and *XZKD* mean the length and width of the line, while *DLMJ* is the area of the line to be subtracted.

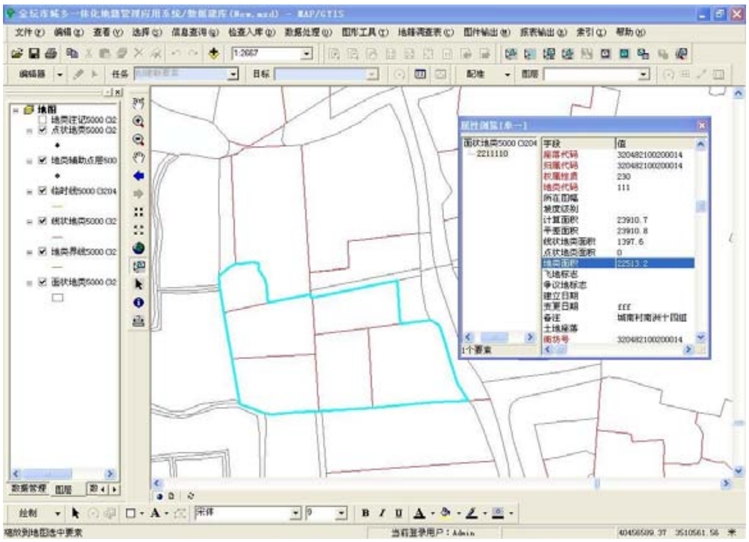

Figure 6. A program developed to calculate the actual area of land use parcels

T_■ZDLTP 查询 : 选择查询

DKH	XZBH	XZCD	XZKD	DLMJ
7	17	8.51	.3	2.6
7	19	.21	1.75	.4
7	17	20.06	.3	6
7	17	2.96	.6	.9
7	9	30.1	.35	10.5
7	8	55.61	3.6	199.8
7	8	36.02	3.6	129.3
7	5	29.66	1.1	32.6
7	5	31.22	2.2	68.7
7	5	44.37	1.1	154.6

记录: 638 共有记录数: 22819

Figure 7. The records of metric relations and detailed topological relations between the parcels and linear features

5. Summary

Spatial relation is so much important for GISs and its applications in reality that more people and academic conferences pay attention to it. In earlier years, most studies focused on simple and perfect spatial features, but fell short of complex objects and can hardly represent complex spatial relations. Someones attempted to propose more complex models but it made things even complicated. In this paper, we suggest a serial model which is composed by some single expression (e.g. 9IM) in details, and integrates topological, metric, directional relations together to describe more exactly. However, this model is unshaped at present and needs to be promoted in future.

6. Acknowledgement

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